$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$AA' \neq A'A \,?$$

$$AA' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\underbrace{4, 1, 1}$$

$$A'A = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda = (\underbrace{4, 1, 1}, 0)$$

Teo 9.1 (Gruber, 2014)

Os autovalores de $X'X$ e XX' não nulos são idênticos.

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ Calcular a SVD.

$$\frac{15-\sqrt{209}}{2} \qquad \frac{15+\sqrt{209}}{2}$$

$$X'X = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & 4 \\ 4 & 2 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$|X'X - \lambda I| = 0 \Rightarrow \begin{vmatrix} 12-\lambda & 4 & 4 \\ 4 & 2-\lambda & 1 \\ 4 & 1 & 2-\lambda \end{vmatrix}$$

$$= (12-\lambda)(2-\lambda)^2 + 16 + 16 - 16(2-\lambda) - (12-\lambda) - 16(2-\lambda)$$

$$= \qquad + \cancel{32} - \cancel{32} + 16\lambda - 12 + \lambda - 32 + 16\lambda$$

$$= (12-\lambda)(2-\lambda)^2 + 33\lambda - 48 = \ldots =$$

$$= -\lambda^3 + 16\lambda^2 - 19\lambda + 4 = 0 \Rightarrow -1(\lambda^3 - 16\lambda^2 + 19\lambda - 4) =$$

$$= (\lambda-1)(\lambda^2 - 15\lambda + 4) = 0$$

$$= -(\lambda-1)\left(\lambda - \frac{15+\sqrt{209}}{2}\right)\left(\lambda - \frac{15-\sqrt{209}}{2}\right) = 0$$

$$14{,}72 \qquad 0{,}27$$

$$\Lambda = \begin{bmatrix} \frac{15+\sqrt{209}}{2} & & \\ & 1 & \\ & & \frac{15-\sqrt{209}}{2} \end{bmatrix} \qquad x_1 = \begin{bmatrix} \frac{9+\sqrt{209}}{8} \\ 1 \\ 1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} ; \qquad x_3 = \begin{bmatrix} \frac{9-\sqrt{209}}{8} \\ 1 \\ 1 \end{bmatrix}$$

$$A_{4\times 3} = P_{4\times 3}\, \Lambda_{3\times 3}\, \Phi^T_{3\times 3}$$

$$= P \cdot \begin{bmatrix} \sqrt{\frac{15+\sqrt{209}}{2}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{\frac{15+\sqrt{209}}{2}} \end{bmatrix} \begin{bmatrix} \frac{9+\sqrt{209}}{8} & 1 & 1 \\ 0 & -1 & 1 \\ \frac{9-\sqrt{209}}{8} & 1 & 1 \end{bmatrix}$$

$$A = P\Lambda^{1/2}\Phi^T$$

$$A\Phi = P\Lambda^{1/2}$$

$$A\Phi\Lambda^{-1/2} = P$$

$$(*) \quad P = \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{4\times 3} \begin{bmatrix} \frac{9+\sqrt{209}}{8} & 0 & \frac{9-\sqrt{209}}{8} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{\frac{15+\sqrt{209}}{2}}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{\frac{15-\sqrt{209}}{2}}} \end{bmatrix}$$

Conta feita na aula

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$ i é não neg. def.

$$L = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}$$

$$l_{11} = \sqrt{a_{11}}$$

$n=2$

$$l_{21} = a_{21}/l_{11} = a_{21}/\sqrt{a_{11}}$$

$i=2$

$$l_{22} = \sqrt{a_{22} - \sum_{k=1}^{1} l_{2k}^2} = \sqrt{a_{22} - l_{21}^2}$$

$$L = \begin{bmatrix} \sqrt{a_{11}} & 0 \\ \dfrac{a_{21}}{\sqrt{a_{11}}} & \sqrt{a_{22} - l_{21}^2} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Conta feita na aula

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$i = \cancel{n}\ 1$

$$l_{11} = \sqrt{a_{11}}$$

$$l_{21} = a_{21}/\cancel{l_{11}} = a_{21}/\sqrt{a_{11}}$$

$$l_{31} = a_{31}/l_{11} = a_{31}/\sqrt{a_{11}}$$

$i = 2$

$$l_{22} = \sqrt{a_{22} - \sum_{k=1}^{1} l_{2k}^2} = \sqrt{a_{22} - l_{21}^2}$$

$j = 3\ (i=2)$

$$l_{32} = \frac{1}{l_{22}}\left(a_{32} - \sum_{k=1}^{1} l_{2k} \cdot l_{3k}\right)$$

$$l_{32} = \frac{1}{l_{22}}\left(a_{32} - l_{21} \cdot l_{31}\right)$$

$i = 3$

$$l_{33} = \sqrt{a_{33} - \sum_{k=1}^{2} l_{3k}^2} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}$$